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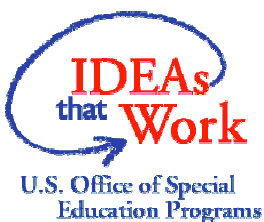
Elementary & Middle Schools
Technical Assistance Center

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EMSTAC Extra

Answering the question.....

How can I teach mathematics to students who have cognitive and behavioral disabilities and who come from diverse cultural and language backgrounds?



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“For people to participate fully in society, they must know basic mathematics. Citizens who cannot reason mathematically are cut off from whole realms of human endeavor.”

(National Research Council, 2001), p. 1)

Our society has achieved enormous advances in science, mathematics and technology over a relatively short period of history. Schools must, then, rise to meet the challenge of preparing an increasingly diverse new generation to compete in this rapidly changing environment – a world that will require large numbers of technologically skilled individuals. A strong foundation in mathematics will be essential to these future professionals.

A number of our Linking Agents have indicated that schools in their districts have an increasingly high number of students from diverse cultural and language groups, some of whom have been identified with cognitive and behavioral disabilities. We refer to such students as culturally, linguistically, and ability diverse (CLAD).

The question arises, then, as to which instructional strategies in mathematics are most effective for this population. This *EMSTAC Extra* will address issues faced by students with disabilities in general as well as unique challenges for students who are clad. We will then explore some strategies that may be useful in the classroom.

What kinds of specific challenges do my students who are clad face in accessing mathematics instruction?

Two general areas of difficulty are of primary concern for students who are clad. These include disability-related challenges, and challenges related to language and cultural issues.

Challenges related to disability

Students with cognitive and behavioral disabilities may experience difficulties in any of a number of areas, including *visual and auditory perceptual, spatial/temporal, memory, and language-related* disabilities. Many students experience a combination of one or more of these challenges, and some also have been identified with attention deficit/hyperactivity (ADD/ADHD) and/or behavioral disorders.

Table 1 illustrates some of the specific difficulties these students may encounter in the mathematics classroom. Sonia, a student with visual perceptual problems, for example, may tend to lose her place on the page, lose track of where she is in a multiple-step problem, and be unable to locate the information she needs on worksheets or textbook pages. If her figure/ground problems are auditory in nature, she may be unable to perceive counting patterns. You may notice that unlike other students in your class, she does not readily pick up counting by two's and five's, a precursor to concepts of multiplication.

Ahmed, meanwhile, is having trouble hearing the difference between the numbers 13 and 30, 14 and 40, and so on. He may even correctly say 40 and think 14! This auditory discrimination problem overlaps difficulties encountered by students who speak languages with phonetic systems that differ from English, which also can prevent them from distinguishing number endings.

Seiki's trouble comes when it is time to read numbers aloud. In contrast to Ahmed, she may say 14 when reading the number 41. Again, this problem is just as common for English language learners and is really quite understandable when you think about the order of the syllables in the English word 'four-teen' and the rest of the teens (shouldn't the four come first?). But Seiki's visual discrimination problem leads her also to reverse her numerals when writing, a problem that should have disappeared in the early grades in a child with normal cognitive functioning. Her disability makes it hard for her to distinguish pennies from dimes, and she seems to get quite lost when trying to learn to tell time from an analog clock. Some of the most serious manifestations of these difficulties may emerge in later grades, as mathematics instruction moves from the concrete to the abstract.

A fourth student, Chun, never seems to remember which is his right sneaker and which is his left. His spatial/temporal difficulties make it difficult for him to master addition and subtraction with regrouping, although he can demonstrate his conceptual understanding with manipulatives. When taking timed tests, he lingers too long on the first problems and is surprised when the bell rings signaling the end of the exam. He avoids word problems at all costs because he finds them confusing and cannot figure out which operation to use, nor which information in the problem is relevant, nor the logical sequence of steps for solving the problem.

Another small group of students seem generally uninterested during math class. They have trouble staying on task, and when encountering even mild frustration during independent work, they seem to give up easily. These attention deficits may make it particularly difficult for students to persist in solving the kinds of open-ended problems recommended by the National Council of Teachers of Mathematics (NCTM)¹(Maccini & Gagnon, 2002).

¹ The National Council of Teachers of Mathematics is a 100,000 member organization that works to promote a high quality mathematics education for all students.

Table 1. Examples of mathematics challenges related to learning disabilities (Bley, 2001)

figure/ground	visual discrimination
<ul style="list-style-type: none"> loses place on page, skips parts of problems cannot locate relevant information on page auditory: cannot perceive counting patterns, trouble skip-counting 	<ul style="list-style-type: none"> may misread numbers writes reversals (especially 2,3,5,6,9) and 13 for 31 etc. trouble recognizing coins, telling time difficulty increases as math moves from concrete to abstract symbols
auditory discrimination	spatial/temporal
<ul style="list-style-type: none"> cannot perceive number endings (e.g., 60 vs. 16) may say numbers correctly but misperceive what is heard 	<ul style="list-style-type: none"> trouble locating position in space difficulty regrouping/combining like terms does not grasp concept of time cannot follow multi-step computation and word problems

Language and cultural challenges

Culturally and linguistically diverse students (with or without disabilities) may encounter challenges in a variety of areas related to learning mathematics. These include issues related to the *language* of mathematics; differences in *cultural background knowledge*; problems related to lack of development in English and native language *reading ability*; and *vocabulary*. *Word problems*, which can be difficult even for the average child, present particularly difficult cognitive and linguistic demands for students who are clad.

Language. Children who are learning English consistently score lower than native English speakers on English-language tests of computation and word problems. This problem may be related to several challenges faced by these children:

- It takes longer to decode² in a second language, which puts an added burden on short-term memory (a person can only hold so much new information in that part of the brain at one time) (Macnamara, 1966)
- Students may have the conceptual knowledge and lack the language; any English-language test is as much a test of English as it is a test of content (Cocking & Chipman, 1988; LaCelle-Peterson & Rivera, 1994).

Like many students, Hung Li has difficulty translating words into mathematical sentences. When provided the sentence “There are three times as many girls as boys,” Hung Li wrote $3g = b$. This problem emerges because the order and syntax of the English sentence can mislead students into literally translating words to numbers and symbols and prevent them from correctly interpreting the conceptual meaning (Cocking & Chipman, 1988). Unfortunately, conducting this exercise in the native language is not always sufficient to resolve the problem, because students need additional instruction in order to conceptualize the mathematics statement correctly.

² Decoding is the process of converting letters and symbols into sounds and meaning during reading.

In addition, research indicates that distracters³ in word problems can cause problems for any child. Students with reading difficulties or mental impairment often have difficulty distinguishing essential from non-essential information. This is particularly true of word problems requiring subtraction. In one study, even presenting the word problem in students' native language did not help much when the problem had distracters (Leon, 1994).

Cultural differences in mathematics. While many people believe that mathematics is universal, a look at mathematical practices from around the world indicates that there are substantial differences in the ways various cultures approach math. Students who have attended school outside of the U.S. may have learned to perform mathematical procedures in a different order, to place numbers differently on the page, or to use symbols that differ from those customary in this country. These differences may be misinterpreted as a lack of mathematical understanding. However, in many cases, students who come from different cultures may have learned different ways to conduct and think about math. Teachers who are unaware of these differences may believe that the child has made a mistake or does not understand the instruction.

$$\begin{array}{r}
 126 \) \ \underline{3} \\
 06 \ \ 42 \\
 0
 \end{array}$$

Figure 1. Mexican division algorithm

For example, many teachers have noticed that immigrant students who have attended school in their home countries are accustomed to using an entirely different set of algorithms for solving math problems. When his teacher asked Elias, for example, to divide 126 by 42,

he produced the algorithm shown in Figure 1. Note that the concept is the same as the one taught in U.S. schools, but the division

symbol and the placement of the numbers is different, and not all of the steps are shown⁴. The quotient, however, still comes out three (Acosta & Spence, 2001).

Lack of background information. Ms. Crenshaw noticed that every one of her English language learners had difficulty with the same question on the practice test for the district mathematics assessment. Upon questioning her students, she discovered that the test item involved solving a word problem about a lawnmower. Not one of the students in this urban school had ever seen a lawnmower, much less heard its English name. The students' lack of background information, not the concept being tested, was a barrier to comprehension.

Instructional Strategies

One of the best things teachers can do for students who are clad is to raise their expectations of what these children will be able to learn. Teachers with good intentions may hold back students by believing they are protecting them from failure, lowering expectations, and discouraging them from pursuing higher-level math. The most

³ Distracters are extraneous information unnecessary to solving a problem.

⁴ Elias' thinking process goes like this: 120 divided by 40 is 3, so I'll try the quotient 3. 3 times 40 is 120. Subtract mentally, 126 minus 120, write down the answer 06. 2 times 3 is 6. Subtract mentally, 6 minus 6, and write down the answer 0. So the correct quotient is 3 with no remainder.

effective teachers, on the other hand, challenge their students rather than protect them (Cocking & Chipman, 1988).

Just as we do for any child with a disability, whether the child is learning in the first or second language, teachers need to address the individual needs of the student within the learning context. Teachers can begin by looking at the specific challenges this child faces in this situation, and adapting instruction accordingly (Demers, 2001).

There are numerous strategies for assisting children with cognitive and behavioral disabilities to learn mathematics. For the purposes of this *EMSTAC Extra*, we will concentrate on a few of the strategies that can help address the needs of culturally, linguistically and ability diverse students:

- Tapping into multiple intelligences;
- connecting with home culture and prior knowledge;
- balancing cognitive and language demands;
- modifying or adapting existing materials; and
- effectively using small group work.

Tap into multiple intelligences

Many of those who work with children who have learning disabilities have recognized that, despite their disabilities, such children often have unique abilities and ways of understanding and interacting with their world. Schools tend to value and assess only two kinds of intelligence (linguistic and logical-mathematical). Howard Gardner (Gardner, 1983; Harvard Project Zero and President and Fellows Harvard College, 2000), however, has identified several additional intelligences. The multiple intelligence (MI) domains researchers have identified include:

- *Linguistic* (syntax, phonology, semantics, pragmatics)
- *Logical-mathematical* (number, categorization, relations)
- *Musical* (pitch, rhythm, timbre)
- *Spatial* (accurate mental visualization, mental transformation of images)
- *Bodily-kinesthetic* (control of one's own body, control in handling objects)
- *Interpersonal* (awareness of others' feelings, emotions, goals, motivations)
- *Intrapersonal* (awareness of one's own feelings, emotions, goals, motivations)
- *Naturalist* (recognition and classification of objects in the environment)

Use of the multiple intelligences approach has been associated with academic and behavioral improvement for diverse learners, including students with LD and culturally and linguistically diverse students (Harvard Project Zero and President and Fellows Harvard College, 2000). By planning lessons around all eight intelligences, teachers provide multiple means for children to access new knowledge and experience new learning. Schools that employ MI believe “that all children have strengths and can learn; that care and respect among people in the school is essential; that learning should be

exciting, and that hard work by adults is necessary to sustain such an environment”⁵ (Harvard Project Zero and President and Fellows Harvard College, 2000).

Help students make connections with home culture and prior knowledge

Children learn best when they can relate their previous experiences to new learning (Burnette, 1999). Teachers can help make this happen by connecting lessons to everyday life in the child’s home, by encouraging families to become involved in their child’s education, and by building on what children already know.

Mathematics lessons can be adapted to a wide range of cultural contexts. Teachers can start by having children interview their family members about how they use mathematics every day in their own lives. You can also invite family participation at back-to-school nights in which parents can share ideas for multicultural activities.

Writing can be an important step in making connections. Having students talk and write about concepts in small groups helps with language acquisition and increases understanding.

Ideas for connecting with home culture:

- Bring family members into the classroom to make presentations about a hobby, to tell a story, or to demonstrate how to make something involving mathematics
- Have children bring objects from home (e.g., ingredients for a family recipe) that could be used to practice math
- Make bulletin board displays of math concepts using magazine photographs from students’ cultures
- Ask students to find songs from their culture involving math and numbers

Balance cognitive and language demands

When possible, combine math and language development objectives, but keep one or the other as the central focus for each lesson. This reduces cognitive overload, so that students can focus on one aspect or the other. For example, when teaching content in English, simplify the language so as to make concepts more accessible to students. Then, in a separate lesson, teach the academic language necessary to access authentic textbooks and to prepare for English-language assessments with more complex language.

When *introducing content*, simplify the language, but keep content at the same challenging level. Identify the academic language to teach prior to the lesson. This language review should include not only vocabulary, but grammatical structures and functions (e.g., reading directions, describing, explaining) as well. Also determine the background knowledge that students will need to understand the concept (Will they know what a washing machine is? Have they been to a baseball game? Are they familiar with

⁵ See [Math Labs](#) in the **Resource** section for examples of lessons that incorporate MI.

Halloween?). Use plenty of visuals, models and demonstrations. Graphic organizers (see Figure 3) can be particularly helpful to students who are clad (LAB at Brown University, 2001).

Increase the language demands once the concept has been mastered. It is particularly important to build students' confidence by providing them with strategies to approach problems written in the complex language found on most standardized assessments. Teach your students that it is not necessary to understand every word in order to solve the problem. You might start by modeling a think-aloud to demonstrate your thinking process as you approach a new problem. Then provide students with a set of sample test problems and have them work together in groups to solve them.

Many school districts are now working on incorporating ESL objectives⁶ into state or local standards. If your district has not yet developed such a curriculum, you can integrate the ESL objectives into your own lesson plans. If you are a bilingual teacher or have a bilingual aide, be sure to teach correct terminology in students' native language. Sometimes aides may not have had training in mathematics; they will need to learn these terms.

Modify or adapt existing materials

To help make materials more accessible for students who are learning English, teachers can modify or adapt existing materials. By getting to know your students and their home cultures, you can discover examples from their everyday lives for use in adapting materials such as word problems.

The word problem in Example A (below) would be very difficult for a CLAD student because a) it requires background information many students may not have (baseball, Giants, innings, etc.); b) it contains too many distracters; c) students may lack the vocabulary to understand the problem; and d) information is too scattered for students with figure/ground or spatial/temporal difficulties. Example B shows one way such a problem could be modified *without reducing the level of mathematics challenge*. Note also that a graphic (the menu) has been added to help students locate the information they need; vocabulary has been simplified; and distracters removed. Remember that, no matter how simplified the language, it is always a good idea to review first to be sure students are familiar with the theme and the vocabulary.

⁶ See the ESL standards at www.tesol.edu/assoc/k12standards/it/01.html.

Figure 2: Modification of a word problem

<p><i>Example A (Original Problem)</i></p> <p>Giant Fans Step Up to the Plate*</p>	<p><i>Example B (Modified Problem)</i></p> <p>At the Soccer Game</p>																			
<p>Your aunt has invited you to the big game! During the 3rd inning J.T. Snow hit a ball into the crowd that almost hit the peanut vendor up in section 21. As the vendor signaled he was okay, you think buying peanuts sounds like a plan. Your aunt has given you a \$20 bill to spend. A local newspaper had mentioned peanuts costing \$1.75, a foot-long Giants dog at \$2.75 and a large souvenir drink at \$3.75. Also mentioned were something known as garlic fries. It's garlic French fries! Gosh if everyone had some of those fries think of the stadium aroma!</p>	<p>You and your friend Carlos are at the soccer game. Your favorite team is winning and you are hungry. You have \$20 to spend. Look at the sign to answer the questions below.</p>																			
	<table border="1"> <thead> <tr> <th data-bbox="799 607 938 680">Snacks</th> <th data-bbox="938 607 1077 680"></th> <th data-bbox="1077 607 1216 680">Drinks</th> <th data-bbox="1216 607 1369 680"></th> </tr> </thead> <tbody> <tr> <td data-bbox="799 687 938 761">burrito</td> <td data-bbox="938 687 1077 761">\$4.00</td> <td data-bbox="1077 687 1216 761">small</td> <td data-bbox="1216 687 1369 761">\$2.25</td> </tr> <tr> <td data-bbox="799 768 938 842">taco</td> <td data-bbox="938 768 1077 842">\$2.75</td> <td data-bbox="1077 768 1216 842">medium</td> <td data-bbox="1216 768 1369 842">\$2.95</td> </tr> <tr> <td data-bbox="799 848 938 920">chips</td> <td data-bbox="938 848 1077 920">\$1.75</td> <td data-bbox="1077 848 1216 920">large</td> <td data-bbox="1216 848 1369 920">\$3.75</td> </tr> </tbody> </table>				Snacks		Drinks		burrito	\$4.00	small	\$2.25	taco	\$2.75	medium	\$2.95	chips	\$1.75	large	\$3.75
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burrito	\$4.00	small	\$2.25																	
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chips	\$1.75	large	\$3.75																	
<p><i>Activity:</i></p> <ol style="list-style-type: none"> Garlic French fries are \$4.00. If you and your Aunt both have garlic fries with a souvenir soft drink, how much would this cost? How many Giants dogs would it take laid end-to-end to "run" around the bases? (To each base is 30 yards.) How much money is left of that original \$20 dollar bill? 	<ol style="list-style-type: none"> You want to buy 2 burritos and 2 large drinks. How much will this cost? It will cost \$ _____ How much money is left from your \$20? \$ _____ is left. How many bags of chips can you buy with the remainder? I can buy _____ bags of chips with the remainder. 																			
<p>*(adapted from Uhrenholt, 1999)</p>																				

Effectively use small group work

In many classrooms, the teacher does most of the talking. But this traditional whole class teacher-lecture style can be problematic for students who are clad because there is not enough opportunity for students to develop communication skills. Small group work provides more chances for students to practice communication through analyzing, discussing and problem-solving. It can increase both engagement and opportunities to learn for diverse groups of students (Vaughn, Hughes, Moody, & Elbaum, 2001). In addition, students from other cultures often feel more comfortable speaking in small groups than they would in a large class format, and have been observed to demonstrate

understanding of mathematical concepts in small groups even when they could not yet do so in a large class format (Brenner, 1998).

Collaborative reading in small groups may be particularly appropriate for work with word problems because students who are clad are encouraged to apply relevant background knowledge in cooperation with other group members (Bos & Vaughn, 1998). This cooperative process increases students' ability to construct meaning and solve problems. Effective cooperative learning⁷ is much more than simply placing students into groups. The following principles may be useful:

- Responsibility for learning rests with the students, not with the teacher.
- Groups are provided the task of exploring meaning, working through a process, and solving problems through consensus, without outside help.
- Each group member is given a clear role.

Cooperative Problem-Solving

Many students have trouble remembering the steps for solving word problems. One way to help them visualize each step is to start with groups of five students and present them with five problems.

- Give each student a different role: e.g., explaining the problem; demonstrating how to address it; working through the problem; checking the solution and stating the answer.
- Dividing up the roles helps students conceptualize the steps to problem-solving; working together in groups provides support when a student gets stuck (Cocking & Chipman, 1988).
- Teach students that this five-step problem-solving method is applicable to every word problem.

Conclusion

Students who are culturally, linguistically and ability diverse can achieve to high standards in mathematics. However, simply placing a student in a general education classroom with the same curriculum as other children does not qualify as equitable opportunity, because the curriculum may not be accessible to students with disabilities nor to children who are culturally and linguistically diverse. To assure access, students need a challenging curriculum as well as instruction that is adapted to their special needs. Teachers of students who are clad need an understanding not only of the child's disability but also of issues of language acquisition and cultural factors that may affect learning. They also need strategies for addressing those needs. Despite the many challenges, teaching CLAD children can be an eye-opening experience. In the words of one teacher, "I think I learned as much this year from my students as they did from me."

⁷ For more information on instructional grouping, current Linking Agents may log in to the private side of the EMSTAC web page (www.emstac.org). Select the Reading Topic, Executive Summary 2, "The Effect of Instructional Grouping Format on the Reading Outcomes of Students with Disabilities: A Meta-Analytic Review."

Web Resources

LAB at Brown University. Northeast and Islands Regional Educational Laboratory. www.lab.brown.edu/. The LAB promotes educational change to provide all students equitable opportunities to succeed. It advocates for populations whose access to excellent education has been limited or denied.

Layered Curriculum. www.help4teachers.com/math.htm Based on current brain-imaging information, Layered Curriculum is a student-centered teaching method, which uses a triangular-shaped model of 3 layers, each requiring a higher level of understanding.

National Clearinghouse for English Acquisition and Language Instruction Educational Programs. <http://www.ncela.gwu.edu/>. See <http://www.ncela.gwu.edu/library/curriculum/index.htm> for articles on mathematics instruction for English language learners.

Project SUMIT www.pzweb.harvard.edu/SUMIT/Default.htm. The Project on Schools Using Multiple Intelligences Theory (SUMIT) is a three-year national investigation of schools using Howard Gardner's theory of multiple intelligences (MI).

Math Labs www.pzweb.harvard.edu/SUMIT/math.htm. The theory of multiple intelligences is applied to an engaging, hands-on format for teaching basic math skills and concepts to students working in small groups.

Tools for Understanding www.ups.edu/community/tofu/. This site is intended for intermediate and middle-grade teachers (particularly those who teach remedial math classes) who teach mathematics and are interested in integrating common technologies into their daily instruction.

Print Resources

Crandall, J., Spanos, G., Christian, D., Simich-Dudgeon, C., and Willetts, K. (1987). Integrating language and content instruction for language minority students. NCBE Teacher Resource Guide Series, September(4). www.ncbe.gwu.edu/ncbepubs/classics/trg/04integrating.htm

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